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MJC 04 sem III B.A./B.Sc. (Hons.)

Differential Equations

Variables separable

Here, the functions of x are linked with dx and the functions of y are linked with dy. Then it can be easily integrated.

Form $\quad F_1(x)\,G_1(y)\,dx + F_2(x)\,G_2(y)\,dy = 0$

$$\Rightarrow \frac{F_1(x)}{F_2(x)}\,dx + \frac{G_2(y)}{G_1(y)}\,dy = 0$$

which can be integrated.

Example $\quad x^2(y+1)\,dx + y^2(x-1)\,dy = 0$

$$\Rightarrow \frac{x^2}{x-1}\,dx + \frac{y^2}{y+1}\,dy = 0$$

$$\Rightarrow \left[\frac{(x^2-1)+1}{x-1}\right]dx + \left[\frac{(y^2-1)+1}{y+1}\right]dy = 0$$

$$\Rightarrow \left[x+1+\frac{1}{x-1}\right]dx + \left[(y-1)+\frac{1}{y+1}\right]dy = 0$$

$$\Rightarrow \int x\,dx + \int dx + \int \frac{dx}{x-1} + \int y\,dy - \int dy + \int \frac{dy}{y+1} = 0$$

which can be easily integrated.

Some equation can be made separable variable in after some transformation, which is clear from the following example.

Q. $\frac{dy}{dx} = (y-4x)^2$

Sol^n It is not possible to separate $f(x)$ from $f(y)$.

So the equation becomes

Let $y-4x = v$

$\Rightarrow \frac{dy}{dx} - 4 = \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = 4 + \frac{dv}{dx}$

$$4 + \frac{dv}{dx} = v^2$$

$\Rightarrow \frac{dv}{dx} = v^2 - 4 \Rightarrow$

It is now separable variable.

$\Rightarrow \frac{dv}{v^2-4} = dx$. Integrating we get-

$\Rightarrow dx - \frac{dv}{v^2-4} = 0$

$\Rightarrow x + \frac{1}{4} \log \frac{v+2}{v-2} = K$

where $v = y-4x$.